(b) Find asymptotic estimates of a fundamental set of solution of the difference equation

$$y(n+2)+\frac{n+1}{n+2}y(n+1)-\frac{2n}{n+2}y(n)=0.$$

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Abel's lemma.

17. Find the general solution of $x(n+1)=Ax(n)$, where $A=\begin{pmatrix}2&2&1\\1&3&1\\1&2&2\end{pmatrix}$.

18. Solve the difference equation $x(n+3)-x(n+2)+2x(n)=0$, $x(0)=1$, $x(1)=1$.

19. State and prove Sturm separation theorem.

20. Find an asymptotic estimate of a fundamental set of solution of $y(n+1)=[A+B(n)]y(n)$, where

$$A=\begin{pmatrix}2&2&1\\1&3&1\\1&2&1\end{pmatrix},\quad B(n)=\begin{pmatrix}1/n^2+1&0&(0.5)^n\\0&(0.2)^n&0\\e^{-n}&0&\log n/n^2\end{pmatrix}.$$

APRIL/MAY 2024

23PEMA25D — DIFFERENCE EQUATIONS

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Prove that the operator Δ^{-1} *is linear.*
2. State the fundamental theorem.
3. Define linearity principle.
4. Write the variation of constants formula.
5. Find the Z-Transform of the sequences $\{n^2a^n\}$.
6. State the convolution theorem.
7. Define oscillatory.
8. Find the limit superior and the limit inferior for the sequences

$S_1 : 0, 1, 0, 1, \ldots\ldots$

$S_1 : 1, -2, 3, -4, \ldots\ldots, (-1)^{n+1}n, \ldots\ldots$

$S_3 : \frac{3}{2}, -\frac{1}{2}, \frac{4}{3}, -\frac{1}{3}, \frac{5}{4}, -\frac{1}{4}, \frac{6}{5}, -\frac{1}{5}, \ldots\ldots$

9. Show that $t^2 \log t + t^3 = o(t^4), t \to \infty$.

10. Define infinite products.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Solve the equation
$x(n+3) - 7x(n+2) + 16x(n+1) - 12x(n) = 0$,
$x(0) = 0$, $x(1) = 1$, $x(2) = 1$.

Or

(b) Solve the difference equation
$y(n+2) + 4y(n) = 8(2^n)\cos\left(\frac{n\pi}{2}\right)$.

12. (a) Solve the system $x(n+1) = Ax(n)$, $x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$,

where $A = \begin{pmatrix} 3/2 & 1/2 & 1/2 \\ 1/2 & 5/2 & -1/2 \\ 0 & 1 & 2 \end{pmatrix}$.

Or

(b) Find A^n if $A = \begin{pmatrix} 0 & 1 & 1 \\ -2 & 3 & 1 \\ -3 & 1 & 4 \end{pmatrix}$.

13. (a) Obtain the inverse Z-transform of
$x(z) = \frac{z(z+1)}{(z-1)^2}$.

Or

(b) Obtain the inverse Z-transform of
$x(z) = \frac{z(z-1)}{(z-2)^2(z+3)}$

14. (a) If $c(n) \geq a(n) > 0$ for all $n > 0$ and $z(n) > 0$ is solution of the equation $c(n)z(n) + \frac{1}{z(n-1)} = 1$, prove that the equation $a(n)y(n) + \frac{1}{y(n-1)} = 1$, has a solution $y(n) \geq z(n) > 1$ for all $n \in Z^+$.

Or

(b) If $b(n)b(n+1) \leq (4-\varepsilon)p^2(n)$ for some $\varepsilon > 0$ for all $n > N$, then every solution of $p(n)x(n+1) + p(n-1)x(x-1) = b(n)x(n)$ is oscillatory.

15. (a) Solve the system $y(n+1) = Ay(n) + g(n)$, where $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$, $g(n) = \begin{pmatrix} n \\ 1 \end{pmatrix}$, $y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Or